(1)

$s=1,2$ $\quad t=-1,0$

$A = \{2s-3t \mid s,t \in \mathbb{Z} \text{ and } 1 \le s \le 2 \text{ and } -1 \le t \le 0\}$

$= \{2(1)-3(-1), 2(1)-3(0), 2(2)-3(-1), 2(2)-3(0)\}$

$= \{2+3, 2, 4+3, 4\}$

$= \{5, 2, 7, 4\}$

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(2)

(a) $A \cap C = \{10\}$

(b) $B - C = \{5, -2, \frac{4}{5}, 0\}$

(c) $C \cup D = \{10, 11, 1, 2\}$

(d) $C \times D = \{(10,10), (10,2), (11,10), (11,2), (1,10), (1,2)\}$

(e) $\mathcal{P}(E) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{1,2,3\}\}$

(3)

(a) $28-(-2)=30$ and $6 \mid 30$

True $\therefore -2 \equiv 28 \pmod 6$

(b)

$$\begin{array}{r} 511 \\ 4\overline{)2045} \\ -20 \\ \hline 04 \\ -4 \\ \hline 05 \\ -4 \\ \hline 1 \end{array}$$

$\overline{2045} = \overline{1}$ in $\mathbb{Z}_4$ is True

(4) See HW 2 #8

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(5) (A) HW 2 #6

(B) Hammock Ch. 8 #15

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(6) (C) HW 3 # 9(c)

(D) See practice test or class notes

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(7)

(E) HW 2 #14(b)

(7) (F)

Let $x \in \{9^n \mid n \in \mathbb{N}\}$.

Then, $x = 9^n$ where $n \in \mathbb{N}$.

So, $x = (3^2)^n = 3^{2n} = 3^k$ where $k = 3n \in \mathbb{N}$.

Thus, $x \in \{3^n \mid n \in \mathbb{N}\}$.

Therefore, $\{9^n \mid n \in \mathbb{N}\} \subseteq \{3^n \mid n \in \mathbb{N}\}$.

Note that $3 \in \{3^n \mid n \in \mathbb{N}\} = \{3^1, 3^2, 3^3, 3^4, \ldots\}$

but $3 \notin \{9^n \mid n \in \mathbb{N}\} = \{9^1, 9^2, 9^3, 9^4, \ldots\}$